Rate of Change of Frequency under line contingencies

Robin Delabays

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R. D., M. Tyloo, and P. Jacquod, *arXiv preprint* **1906.05698** (2019)

Hes·so Valais Wallis
School of Engineering

Where is HES-SO?

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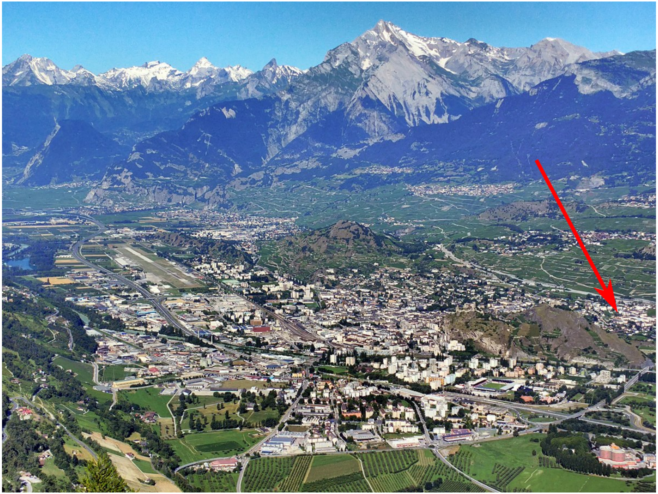
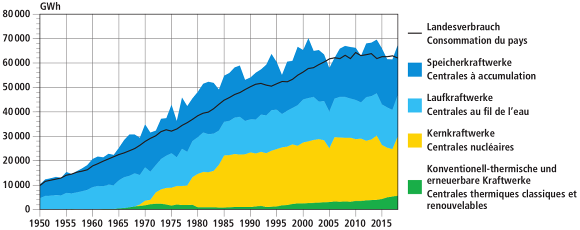


Motivation



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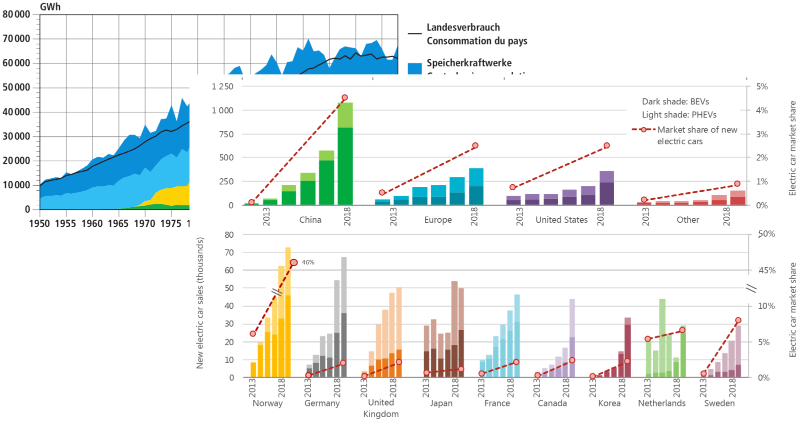

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Motivation

What is the impact of a given contingency?

What are the critical elements in a grid?

How to identify (efficiently) critical operating states?

The Swing Equations

We consider:

$$m_i\ddot{\theta}_i + d_i\dot{\theta}_i = P_i - \sum_j b_{ij}(\theta_i - \theta_j)\,, \qquad i \in \{1, ..., n\}\,,$$

m_i: inertia, d_i: damping, b_{ij}: susceptance, P_i: generation/load.

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$$M\ddot{\boldsymbol{\theta}} + D\dot{\boldsymbol{\theta}} = \mathbf{P} - \mathbb{L}\boldsymbol{\theta}\,,$$

$M = \mathrm{diag}(\mathbf{m})$, $D = \mathrm{diag}(\mathbf{d})$, $\mathbb{L}$ Laplacian matrix.

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Shorthand notation: $\omega_i := \dot{\theta}_i$.

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Analytical solution

Assume $m_i \equiv m$, $d_i \equiv d$, and consider angle deviations

$$\boldsymbol{\delta\theta}(t) = \boldsymbol{\theta}(t) - \boldsymbol{\theta}^*\,, \qquad \boldsymbol{\theta}^* = \mathbb{L}^\dagger \mathbf{P}_0\,, \qquad \mathbf{P}(t) = \mathbf{P}_0 + \boldsymbol{\delta}\mathbf{P}(t)\,.$$

M. Tyloo and P. Jacquod, *Phys. Rev. E* **100** 032303 (2019).

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Expanding on the eigenmodes of $\mathbb{L}$:

$$\mathbb{L}\mathbf{u}^{(\alpha)} = \lambda_\alpha \mathbf{u}^{(\alpha)}\,, \qquad \boldsymbol{\delta\theta}(t) = \sum_{\alpha=1}^{n} c_\alpha(t)\mathbf{u}^{(\alpha)}\,.$$

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$$m\ddot{c}_\alpha(t) + d\,\dot{c}_\alpha(t) = \boldsymbol{\delta}\mathbf{P}(t)\cdot\mathbf{u}^{(\alpha)} - \lambda_\alpha c_\alpha(t)\,, \quad \alpha = 1, ..., n\,.$$

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Analytical solution:

$$c_\alpha(t) = m^{-1} e^{-(\gamma+\Gamma_\alpha)t/2} \int_0^t e^{\Gamma_\alpha t_1} \int_0^{t_1} \boldsymbol{\delta}\mathbf{P}(t_2)\cdot\mathbf{u}^{(\alpha)} e^{(\gamma-\Gamma_\alpha)t_2/2} dt_2 dt_1\,.$$

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Swing Equations, $i \in \{1, ..., n\}$:

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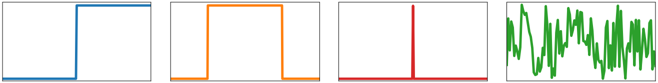

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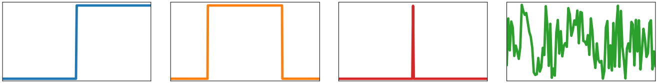
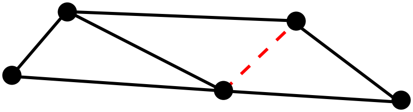
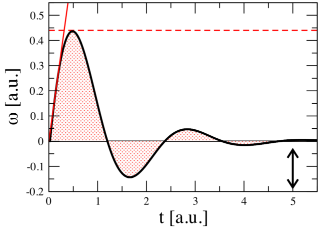


Line perturbations: multiplicative, $\mathbb{L} \to \mathbb{L} - \beta \mathbf{e}_{ij}\mathbf{e}_{ij}^\top$.



Measures of the impact

Transmission losses: $\mathcal{L}_2$-norm of angle deviations.



$$\int_0^\infty \delta\theta^2(t)dt$$

E. Tegling, B. Bamieh, and D. F. Gayme, *IEEE Trans. Control Netw. Syst.* **2** 254 (2015).

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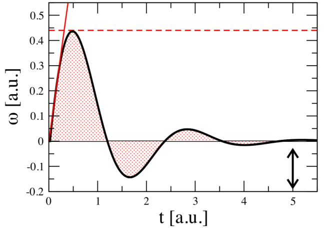
Transmission losses: $\mathcal{L}_2$-norm of angle deviations.

Primary control effort: $\mathcal{L}_2$-norm of frequency deviations.



$$\int_0^\infty \omega^2(t)dt$$

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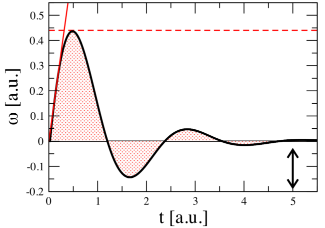
Nadir: $\mathcal{L}_\infty$-norm of frequency deviations.



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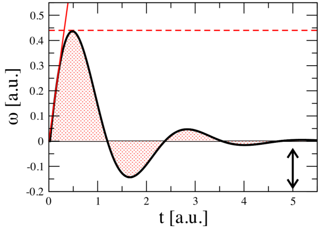
Measures of the impact

Transmission losses: $\mathcal{L}_2$-norm of angle deviations.

Primary control effort: $\mathcal{L}_2$-norm of frequency deviations.

Nadir: $\mathcal{L}_\infty$-norm of frequency deviations.

RoCoF: $\mathcal{L}_\infty$-norm of the time derivative of the frequency.



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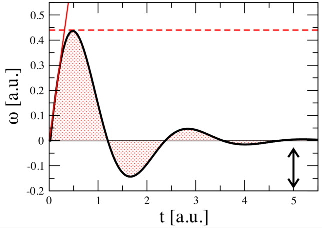
The RoCoF

Maximal local RoCoF:

$$\mathrm{RoCoF} = \max_i \|\dot{\omega}_i(t)\|_\infty \,.$$



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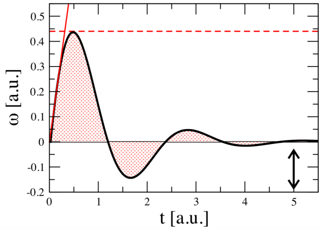

RoCoF is maximal at $t = 0^+$.

$$\boldsymbol{\omega}(0) = 0\,, \qquad \mathbf{P} = \mathbb{L}\,\boldsymbol{\theta}(0)\,, \qquad \mathbb{L}^* = \mathbb{L} - b_{ij}\mathbf{e}_{ij}\mathbf{e}_{ij}^\top\,,$$

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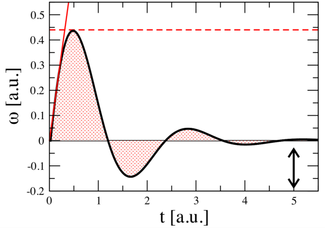

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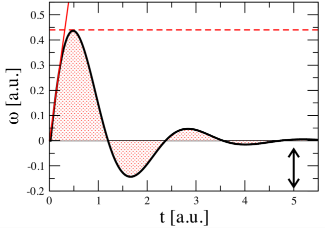

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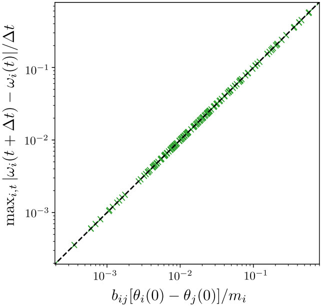
$$\Longrightarrow \; \dot{\omega}_k = (\delta_{ik} - \delta_{jk})\frac{b_{ij}(\theta_i - \theta_j)}{m_k}\,. \quad \rightarrow \text{RoCoF at nodes } i \text{ and } j.$$

Numerics (IEEE 118-Bus)



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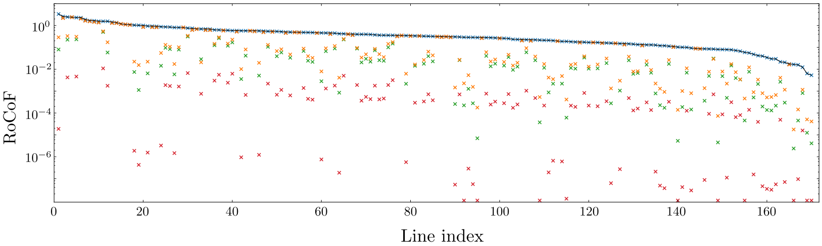

Black line: theory.

x: 100% inertia at loads, RoCoF at all nodes.

x: 100% inertia at loads, RoCoF at generators only.

x: 1% inertia at loads, RoCoF at generators only.

x: 0% inertia at loads, RoCoF at generators only.

Including uncertainties

Statistics on generation and loads:

$$\mathbb{E}\,[P_k] = \mu_k\,, \qquad \mathbb{E}\,[(P_k - \mu_k)(P_\ell - \mu_\ell)] = \Pi_{k\ell}\,.$$

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One gets:

$$\mathbb{E}(\dot{\omega}_i) = \frac{b_{ij}}{m_i}\mathbf{e}_{ij}^\top \mathbb{L}^\dagger \boldsymbol{\mu}\,, \qquad \mathrm{var}(\dot{\omega}_i) = \frac{b_{ij}^2}{m_i^2}\mathbf{e}_{ij}^\top \mathbb{L}^\dagger \Pi \mathbb{L}^\dagger \mathbf{e}_{ij}\,.$$

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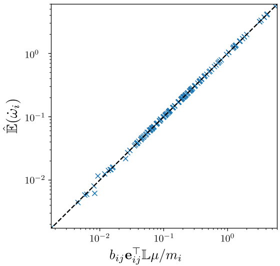

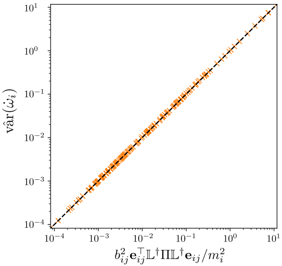

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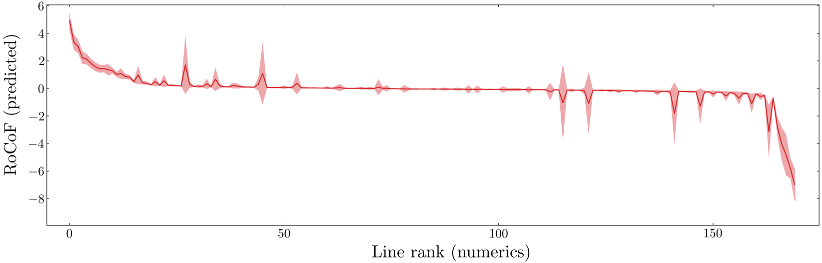

Conclusion

The RoCoF after a line loss is:

- proportional to the flow on the line;
- inversely propotional to the inertia of the node where it is measured.

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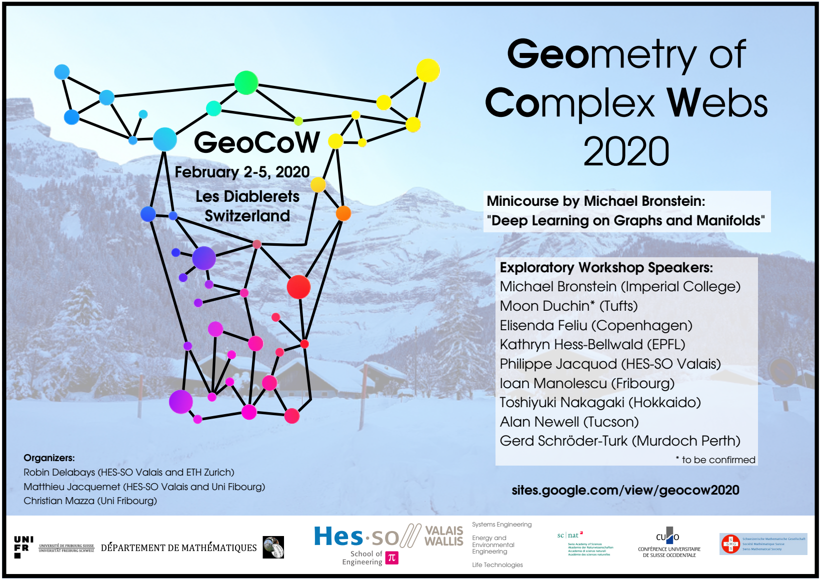
Caveat: We assume inertia at every nodes, which is not true (yet...).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Moon Duchin* (Tufts)
Elisenda Feliu (Copenhagen)
Kathryn Hess-Bellwald (EPFL)
Philippe Jacquod (HES-SO Valais)
Ioan Manolescu (Fribourg)
Toshiyuki Nakagaki (Hokkaido)
Alan Newell (Tucson)
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