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Locating the source of forced oscillations: A system-agnostic approach

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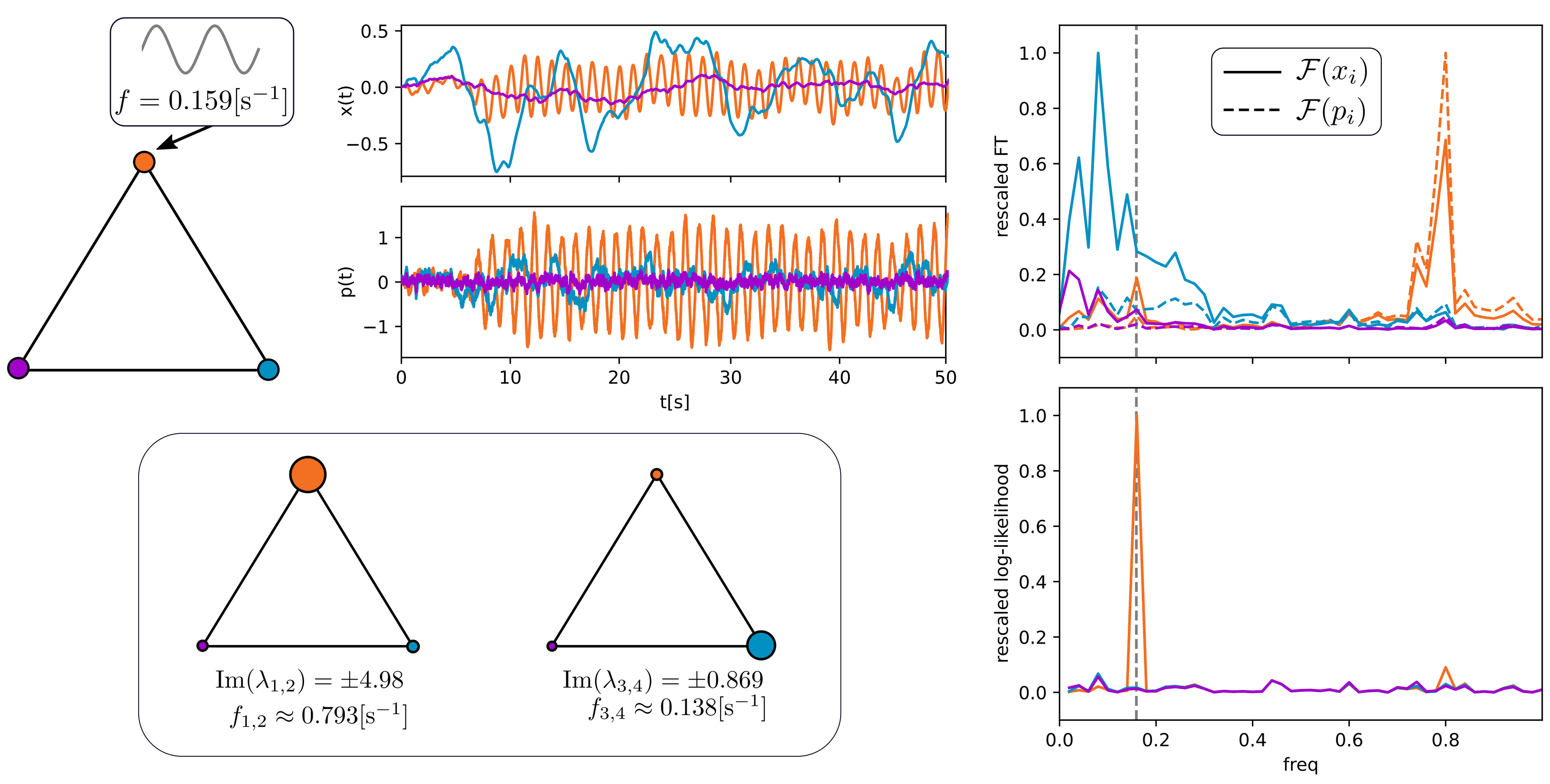
Challenges for locating the oscillation source

The swing equations [1] with forced oscillations at bus l, for $i = 1, ..., n$,

$$m_i\ddot{\theta}_i + d_i\dot{\theta}_i = P_i - \sum_{j=1}^{n} V_iV_j\left[B_{ij}\sin(\theta_i-\theta_j) + G_{ij}\cos(\theta_i-\theta_j)\right] + \delta_{i,l}\gamma\sin(2\pi f t + \phi)\,. \qquad (1)$$

Depending on the characteristics of the forcing, it can be hard to:

- Identify the forcing's frequency $f \in \mathbb{R}$;
- Identify the forcing's location $l \in \{1, ..., n\}$.



Simple example of a forcing that is hard to locate and identify. The star indicates normalization.

Log-likelihood optimization

Linearizing Eq. (1), with unknowns A, l, γ, f, ϕ,

$$dX_t = AX_t dt + \gamma \boldsymbol{e}_l \mathrm{Re}\left(e^{2\pi i(ft+\phi)}\right)dt + dW_t\,, \qquad A = \begin{pmatrix} \mathbf{0} & I_n \\ M^{-1}Y & M^{-1}D \end{pmatrix} = \begin{pmatrix} \mathbf{0} & I_n \\ Y_M & [d_M] \end{pmatrix}\,, \qquad (2)$$

which is discretized as

$$\Delta_{t_j} = AX_{t_j} + \gamma\boldsymbol{e}_l \mathrm{Re}\left(e^{2\pi i(kj/T+\phi)}\right) + \xi_j\,, \qquad (3)$$

with

$$\Delta_{t_j} = (X_{t_{j+1}} - X_{t_j})/\tau\,, \qquad \tau = T/N\,, \qquad t_j = j\tau\,, \qquad k \approx \nu T\,, \qquad \xi_j \sim \mathcal{N}(0,\tau)\,. \qquad (4)$$

Therefore, the variables A, γ, l, k, ϕ should minimize the likelihood

$$L_0(A,\gamma,l,k,\phi) = \frac{1}{N}\sum_{j=0}^{N-1}\left\|\Delta_{t_j} - AX_{t_j} - \gamma\boldsymbol{e}_l\mathrm{Re}\left(e^{2\pi i(kj/T+\phi)}\right)\right\|^2\,. \qquad (5)$$

A convenient rephrasing of $L_0(A,\gamma,l,k,\phi)$ [2] allows to get rid of ϕ and renders the optimization tractable by the interior point method Ipopt [3].

> The optimization relies solely on times series of voltage angles and frequencies.

In summary, we solve

$$Y_M, d_M, \gamma, l, k = \mathrm{argmin}\; L_0(Y_M, d_M, \gamma, l, k)\,. \qquad (6)$$

Synthetic data

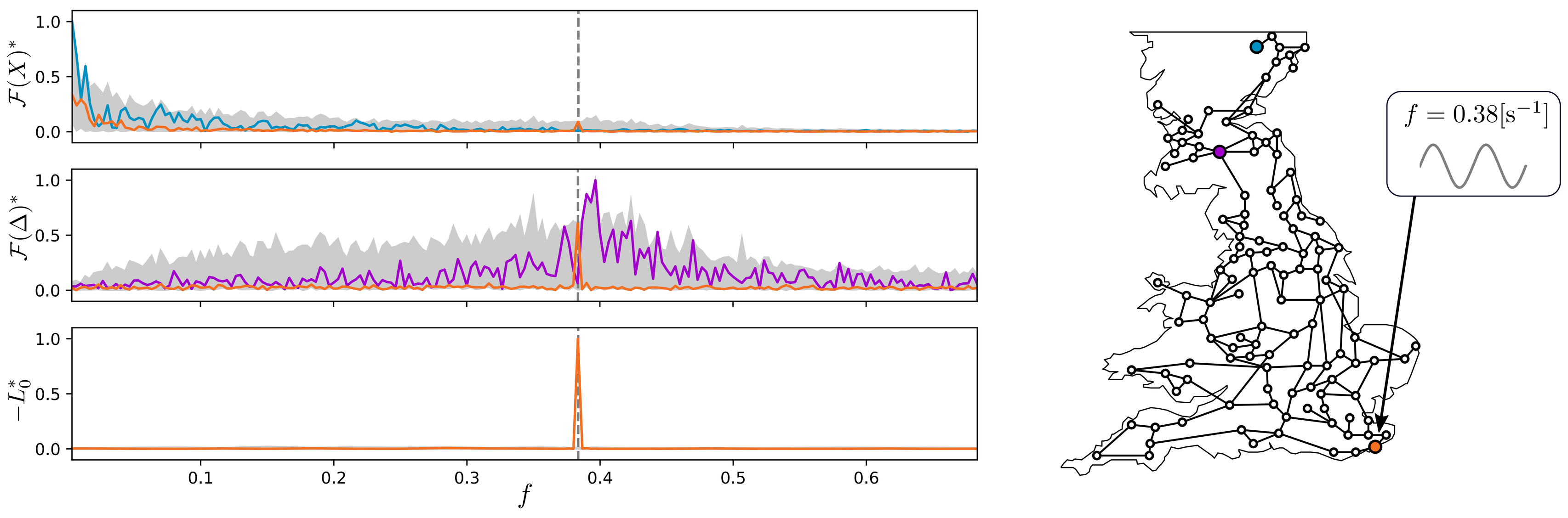
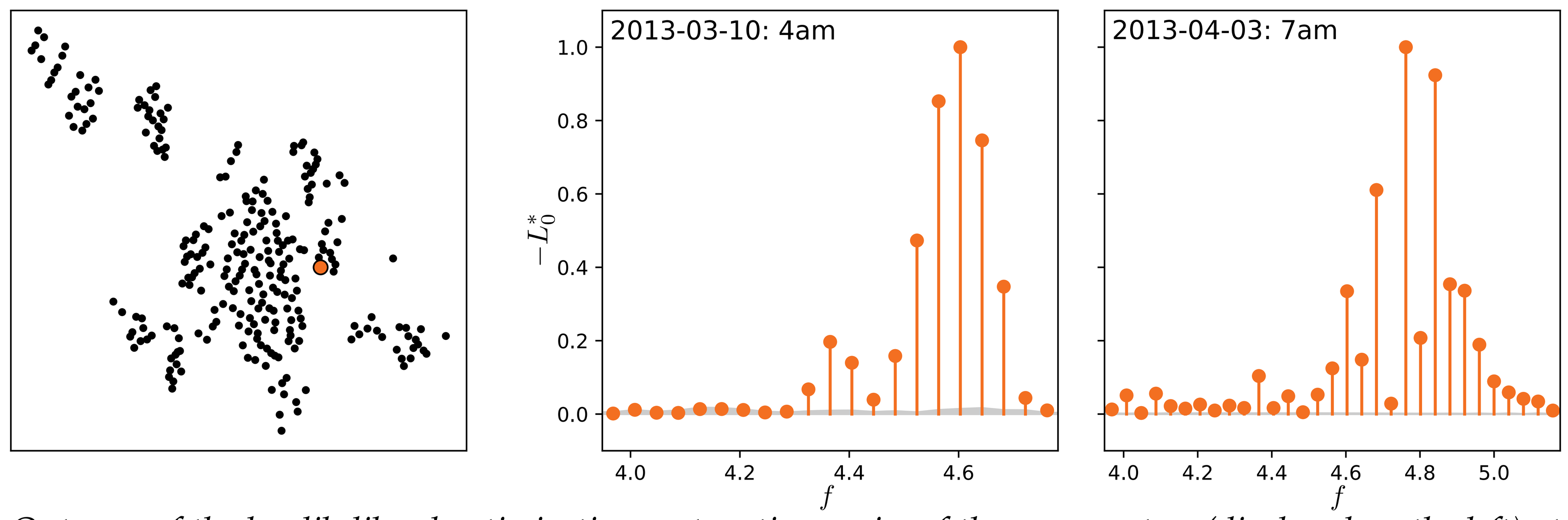
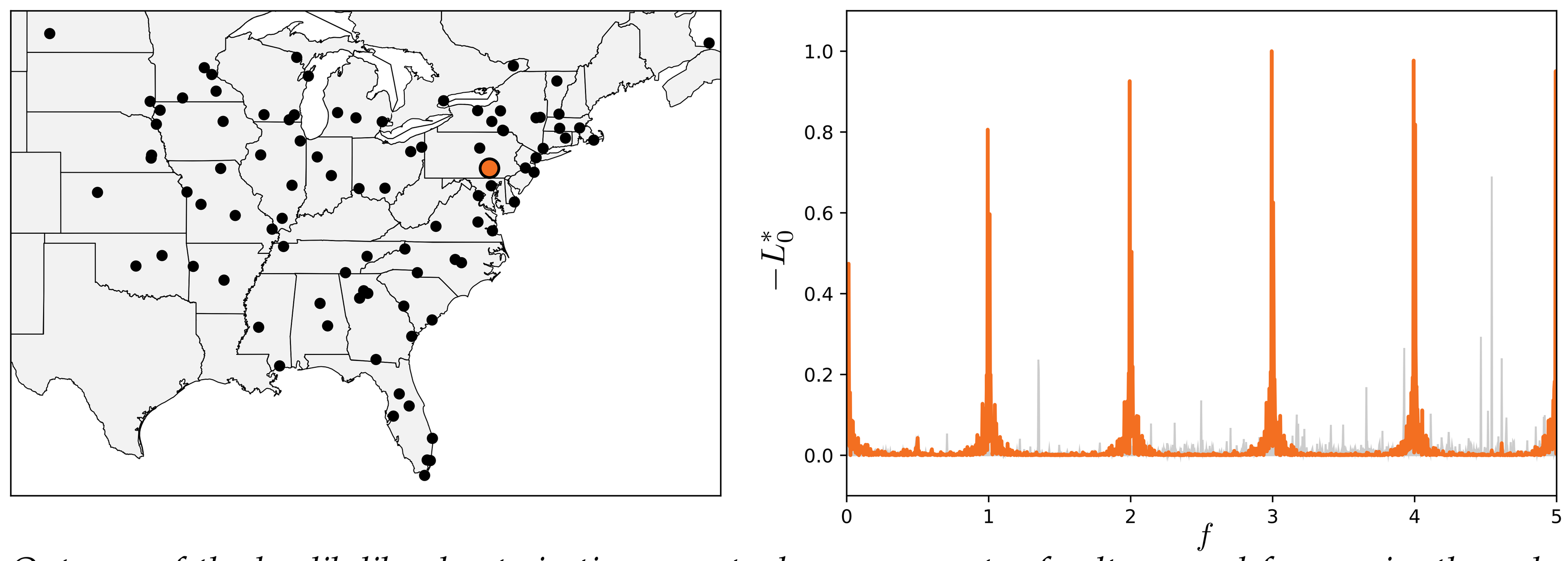


Illustration of the outcome of the optimization of the log-likelihood in on a representation of the UK power grid, together with the Fourier Transforms of the position and velocity signals. The star indicates normalization.

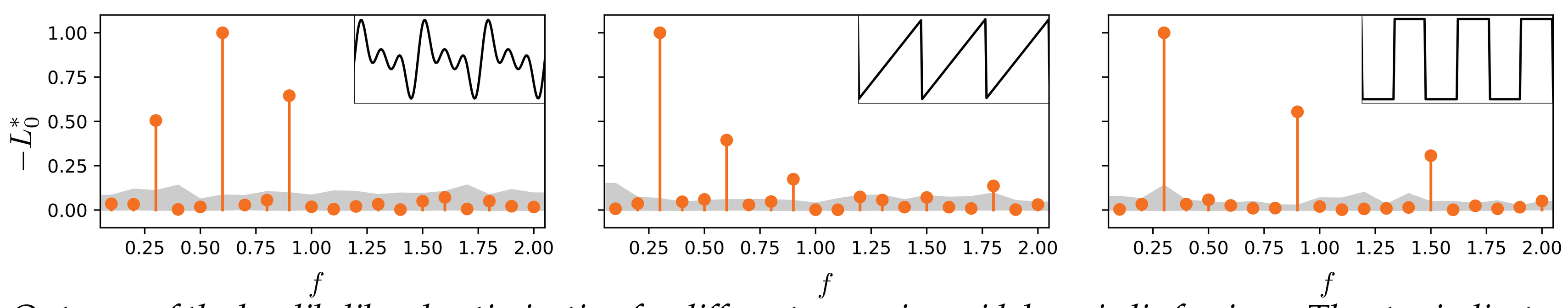
Measurements data



Outcome of the log-likelihood optimization on two time series of the same system (displayed on the left) at two different points in time. Time series are actual measurements, provided by a system operator, the same data were used before in Ref. [4]. Whereas we do not have any ground truth of an actual forcing, we observe consistency of our identification in location and frequency, at two distinct time of measurements. The star indicates normalization.



Outcome of the log-likelihood optmization on actual measurements of voltages and frequencies throughout the US. Data were obtained and processed by the Power IT Lab, at Oak Ridge National Lab and UT Knoxville. Surprising peaks at integer frequencies can be seen, consistently originating from the same bus. The star indicates normalization.



Outcome of the log-likelihood optimization for different, non-sinusoidal, periodic forcings. The star indicates normalization.

Relaxation of the bus index

Taking the forcing's amplitude as a vector in the optimization, the log-likelihood is written as

$$L_1(A,\boldsymbol{\gamma},k,\phi) = \frac{1}{N}\sum_{j=0}^{N-1}\left\|\Delta_{t_j} - AX_{t_j} - \boldsymbol{\gamma}\mathrm{Re}\left(e^{2\pi i(kj/T+\phi)}\right)\right\|^2\,, \qquad (7)$$

and we solve

$$Y_M, d_M, \boldsymbol{\gamma}, k = \mathrm{argmin} L_1(Y_M, d_M, \boldsymbol{\gamma}, k)\,. \qquad (8)$$

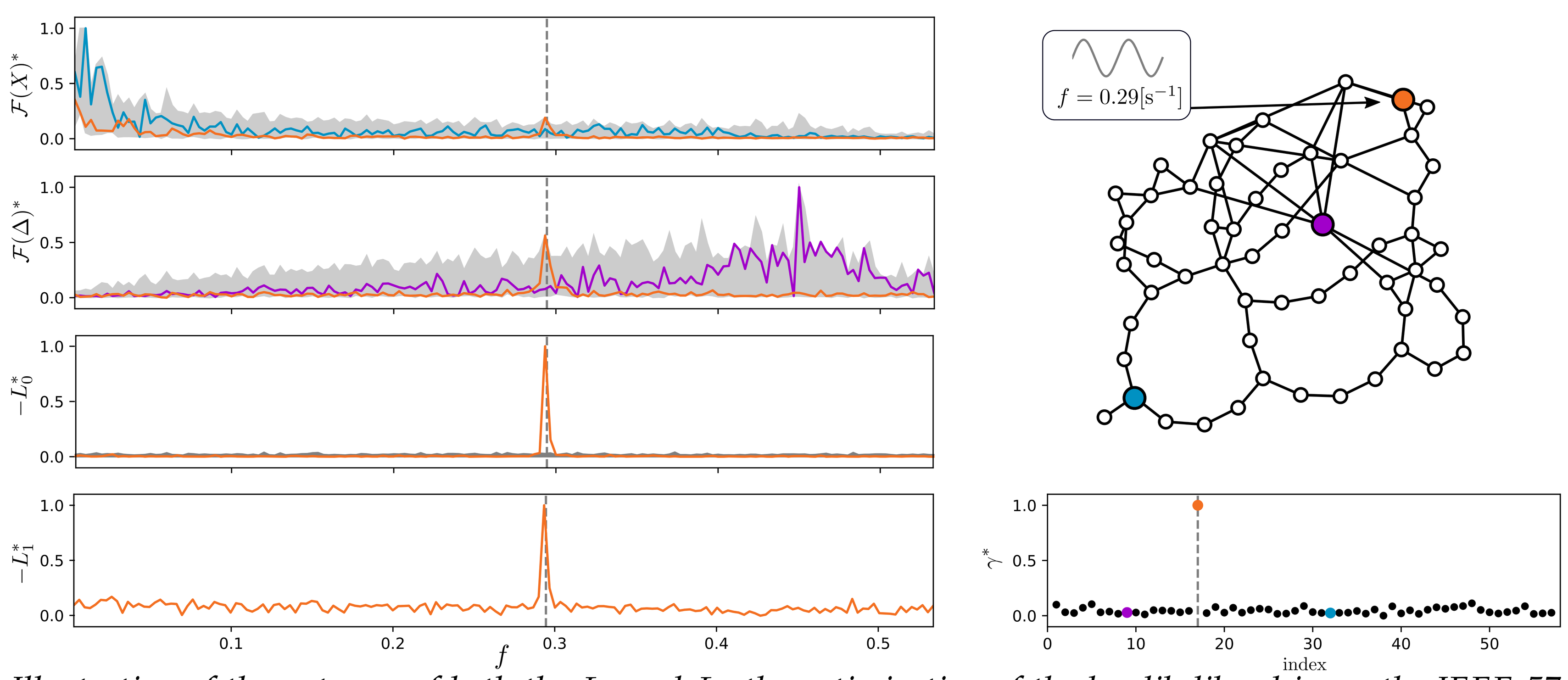


Illustration of the outcome of both the L_0 and L_1 the optimization of the log-likelihood in on the IEEE-57 test case. Compared with the Fourier Transforms of the position and velocity signals. The star indicates normalization.

References

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[4] M. Escobar, D. Bienstock, and M. Chertkov, "Learning from power system data stream", *Proc. of the IEEE PowerTech 2019* (2019). DOI: 10.1109/PTC.2019.8810950

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