Robin Delabays

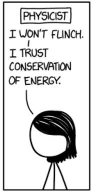
www.delabays.xyz

Hes·so VALAIS WALLIS
School of Engineering

Data-driven network inference: from transmission to distribution grids

Inference of...

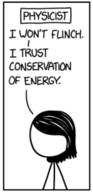
1. Transmission grids



www.xkcd.com/2539

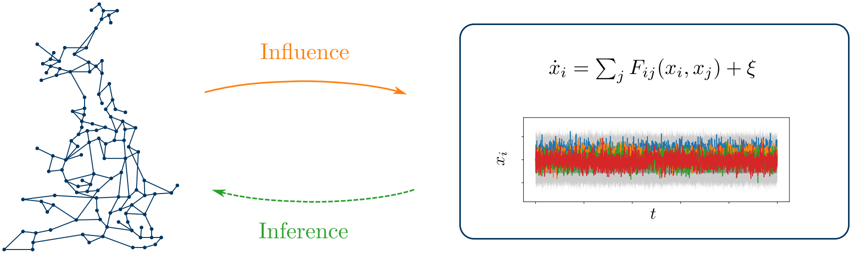
Inference of...

1. **Transmission grids**
2. **Distribution grids**



www.xkcd.com/2539

The graph impacts dynamics... and vice versa



W.-X. Wang, Y.-C. Lai, and C. Grebogi, *Phys. Rep.* **644** (2016).

I. Brugere, B. Gallagher, and T. Y. Berger-Wolf, *ACM Comput. Surv.* **51** (2018).

Various approaches

Probing: [Yu et al., *Phys. Rev. Lett.* **97** (2006)], [Timme, *Phys. Rev. Lett.* **98** (2007)], [Dong et al., *PLoS ONE* **8** (2013)], [Basiri et al., *Phys. Rev. E* **98** (2018)], [Tyloo and D., *J. Phys. Complexity* **2** (2021)], ...

Maximum likelihood/cost minimization: [Hoang et al., *Phys. Rev. E* **99** (2019)], [Makarov et al., *J. Neurosci. Methods* **144**(2005)], [Shandilya and Timme, *New J. Phys.* **13** (2011)], [Panaggio et al., *Chaos* **29** (2019)], ...

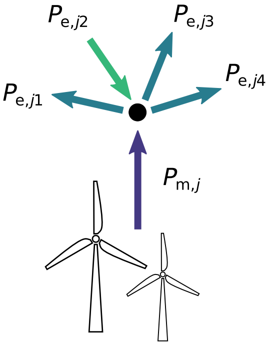
Statistical properties of trajectories: [Dahlhaus et al., *J. Neurosci. Methods* **77** (1997)], [Sameshima and Baccalá, *J. Neurosci. Methods* **94** (1999)], [Ren et al., *Phys. Rev. Lett.* **104** (2010)], [Newman, *Nature Physics* **14** (2018)], [Peixoto, *Phys. Rev. Lett.* **123** (2019)], ...

Power flow equations

Power flow equations:

$$P_j = \sum_k \underbrace{V_j V_k \left[B_{jk} \sin(\theta_j - \theta_k) + G_{jk} \cos(\theta_j - \theta_k)\right]}_{P_{\mathrm{e},jk}}$$

$$Q_j = \sum_k V_j V_k \left[B_{jk} \cos(\theta_j - \theta_k) - G_{jk} \sin(\theta_j - \theta_k)\right]$$



J. Machowski, J. W. Bialek, and J. R. Bumby, *Power System Dynamics*, 2nd ed. (Wiley, Chichester, U.K, 2008).

Transmission grid inference

Melvyn Tyloo
Uni Exeter

Philippe Jacquod
HES-SO Sion

doi.org/10.1063/5.0058739

Simplified problem

$$P_j = \sum_k V_j V_k \left[B_{jk} \sin(\theta_j - \theta_k) + G_{jk} \cos(\theta_j - \theta_k)\right]$$

J. Machowski, J. W. Bialek, and J. R. Bumby, *Power System Dynamics*, 2nd ed. (Wiley, Chichester, U.K, 2008).

Simplified problem

$$P_j = \sum_k V_j V_k \left[B_{jk} \sin(\theta_j - \theta_k) + G_{jk} \cos(\theta_j - \theta_k)\right]$$

In transmission grids (high voltage):

$$V_j \approx \text{constant}$$
$$G_{jk} \approx 0$$
$$P_j = \sum_k B_{jk} \sin(\theta_j - \theta_k)$$

The voltage phases evolve following the mismatch of the latter equation (swing equations).

J. Machowski, J. W. Bialek, and J. R. Bumby, *Power System Dynamics*, 2nd ed. (Wiley, Chichester, U.K, 2008).

Network inference from ambient noise

Considering deviations from the fixed point

$$\mathbf{x}(t) = \boldsymbol{\theta}(t) - \boldsymbol{\theta}^*$$

Network inference from ambient noise

Considering deviations from the fixed point

$$\mathbf{x}(t) = \boldsymbol{\theta}(t) - \boldsymbol{\theta}^*$$

The dynamics is approximated as

$$\dot{\mathbf{x}} \approx \mathcal{J}\mathbf{x} + \boldsymbol{\xi}$$

where

$$\mathcal{J}_{ij} = \begin{cases} B_{ij}\cos(\theta_i^* - \theta_j^*)\,, & i \neq j\,, \\ -\sum_k B_{ik}\cos(\theta_i^* - \theta_k^*)\,, & i = j\,, \end{cases}$$

and $\boldsymbol{\xi}$ is the noise.

Remark: The Jacobian $\mathcal{J}$ is a (weighted) Laplacian matrix, in particular symmetric.

Correlated noise and two-point correlator

Consider a time-correlated noise $\boldsymbol{\xi}$:

$$\mathbb{E}[\xi_j(t)] = 0 \qquad \mathbb{E}[\xi_j(t)\xi_k(t')] = \delta_{jk} e^{-\tau_0|t-t'|}$$

and look at $\mathbb{E}[\dot{x}_j \dot{x}_k]$.

Correlated noise and two-point correlator

Consider a time-correlated noise $\boldsymbol{\xi}$:

$$\mathbb{E}[\xi_j(t)] = 0 \qquad \mathbb{E}[\xi_j(t)\xi_k(t')] = \delta_{jk} e^{-\tau_0|t-t'|}$$

and look at $\mathbb{E}[\dot{x}_j \dot{x}_k]$.

Then for $\tau_0 \ll 1$,

$$\mathbb{E}[\dot{x}_j \dot{x}_k] = \delta_{jk} + \sum_{m=1}^{\infty} (-\tau_0)^m \left(\mathcal{J}^m\right)_{jk}$$

and for $\tau_0 \gg 1$,

$$\mathbb{E}[\dot{x}_j \dot{x}_k] = \frac{1}{n} - \sum_{m=1}^{\infty} (-\tau_0)^{-m} \left(\mathcal{J}^{-m}\right)_{jk}$$

Distribution grid inference

Marc Gillioz
HES-SO Sion

Distribution grid vs. transmission grid

Relevance:

- Line parameters are often not known.
- Sometimes even the structure is not clear.
- Grid operators are interested in accurate models.
- By 2027, 80% of deployment in CH.

Pros:

- Structure is usually simple (tree).

Cons:

- Data quality.
- Actual data availability.

One among many algorithms

Under **"reasonable"** assumptions, Park, Deka, and Chertkov (2018) can estimate the inverse of the resistance and reactance Laplacian matrices: $L_{1/r}^{\dagger}$ and $L_{1/x}^{\dagger}$.

Linear coupled power flow (LC-PF)

$$P_j = \sum_k B_{jk}(\theta_j - \theta_k) + G_{jk}(V_j - V_k) \qquad \mathbf{P} = L_{1/x}\boldsymbol{\theta} + L_{1/r}\mathbf{V}$$

$$Q_j = \sum_k B_{jk}(V_j - V_k) - G_{jk}(\theta_j - \theta_k) \qquad \mathbf{Q} = L_{1/x}\mathbf{V} - L_{1/r}\boldsymbol{\theta}$$

D. Deka, M. Chertkov, and S. Backhaus, IEEE TCNS (2017).

Linear coupled power flow (LC-PF)

$$P_j = \sum_k B_{jk}(\theta_j - \theta_k) + G_{jk}(V_j - V_k) \qquad \mathbf{P} = L_{1/x}\boldsymbol{\theta} + L_{1/r}\mathbf{V}$$

$$Q_j = \sum_k B_{jk}(V_j - V_k) - G_{jk}(\theta_j - \theta_k) \qquad \mathbf{Q} = L_{1/x}\mathbf{V} - L_{1/r}\boldsymbol{\theta}$$

leads to

$$\mathbf{V} = L^{\dagger}_{1/r}\mathbf{P} + L^{\dagger}_{1/x}\mathbf{Q}$$
$$\boldsymbol{\theta} = L^{\dagger}_{1/x}\mathbf{P} - L^{\dagger}_{1/r}\mathbf{Q}$$

D. Deka, M. Chertkov, and S. Backhaus, IEEE TCNS (2017).

Extracting information from correlations

Taking the voltage to power correlations:

$$\mathbb{E}(V_j P_k) = (L^{\dagger}_{1/r})_{jk}\mathbb{E}(P_k^2) + (L^{\dagger}_{1/x})_{jk}\mathbb{E}(P_k Q_k)$$
$$\mathbb{E}(V_j Q_k) = (L^{\dagger}_{1/r})_{jk}\mathbb{E}(P_k Q_k) - (L^{\dagger}_{1/x})_{jk}\mathbb{E}(Q_k^2)$$

S. Park, D. Deka, and M. Chertkov, Proceeding of PSCC (2018)

Extracting information from correlations

Taking the voltage to power correlations:

$$\mathbb{E}(V_j P_k) = (L^\dagger_{1/r})_{jk}\mathbb{E}(P_k^2) + (L^\dagger_{1/x})_{jk}\mathbb{E}(P_k Q_k)$$
$$\mathbb{E}(V_j Q_k) = (L^\dagger_{1/r})_{jk}\mathbb{E}(P_k Q_k) - (L^\dagger_{1/x})_{jk}\mathbb{E}(Q_k^2)$$

All (co)variances can be estimated empirically.

S. Park, D. Deka, and M. Chertkov, Proceeding of PSCC (2018)

Extracting information from correlations

Taking the voltage to power correlations:

$$\mathbb{E}(V_j P_k) = (L^{\dagger}_{1/r})_{jk}\mathbb{E}(P_k^2) + (L^{\dagger}_{1/x})_{jk}\mathbb{E}(P_k Q_k)$$
$$\mathbb{E}(V_j Q_k) = (L^{\dagger}_{1/r})_{jk}\mathbb{E}(P_k Q_k) - (L^{\dagger}_{1/x})_{jk}\mathbb{E}(Q_k^2)$$

All (co)variances can be estimated empirically.

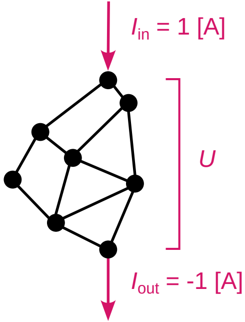
Then one can solve for $L^{\dagger}_{1/r}$ and $L^{\dagger}_{1/x}$.

S. Park, D. Deka, and M. Chertkov, Proceeding of PSCC (2018)

Effective resistance

Hence, we get the **effective resistance** or **resistance distance**:

$$d_r(i,j) = (L^\dagger_{1/r})_{ii} + (L^\dagger_{1/r})_{jj} - 2(L^\dagger_{1/r})_{ij} \,.$$

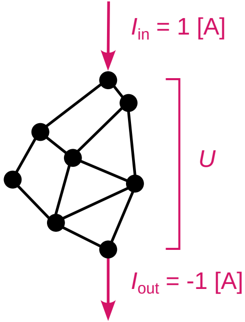


D. J. Klein and M. Randić, J. Math. Chem. **12** (1993)

Effective resistance

Hence, we get the **effective resistance** or **resistance distance**:

$$d_r(i,j) = (L^\dagger_{1/r})_{ii} + (L^\dagger_{1/r})_{jj} - 2(L^\dagger_{1/r})_{ij}\,.$$



As the graph is a tree,

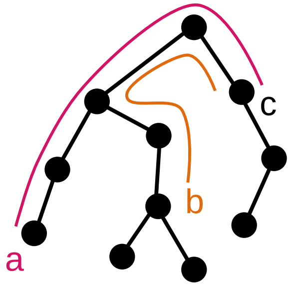
$$d_r(i,j) = \sum_{e\in\mathcal{P}_{ij}} R_e$$

D. J. Klein and M. Randić, J. Math. Chem. **12** (1993)

Recursive grouping algorithm

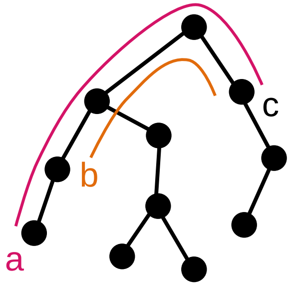
Given pairwise distances $d(a, b)$ and assuming **the graph is a tree**. Compute

$$\Phi_{abc} = d(a, c) - d(b, c)$$



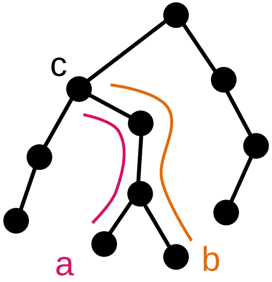
Recursive grouping algorithm: parent-child

If $d(a,b) = \Phi_{abc}$ for all c ,then b is a parent to a.



Recursive grouping algorithm: siblings

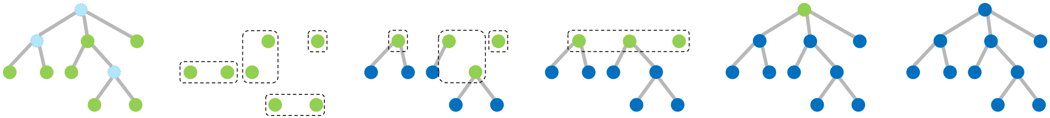
If $\Phi_{abc} = \Phi_{abc'}$ for all c, c', then a and b are siblings.



Then a new parent h is added,

$$d(a,h) = \frac{1}{2}(d(a,b) + \Phi_{abc}) \qquad d(c,h) = d(a,c) - d(a,h)$$

Recursive grouping algorithm



S. Park, D. Deka, and M. Chertkov, Proceeding of PSCC (2018)

Let's run the algo on real data

Semi-synthetic data:

Smart meter measurements from GOFLEX...

https://goflex-project.eu/

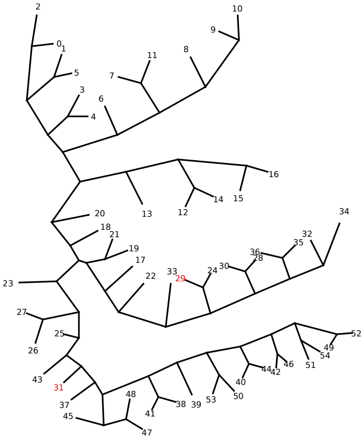
Khan and Hayes, *2022 IEEE PESGM* (2022), doi: 10.1109/PESGM48719.2022.9916806

Let's run the algo on real data

Semi-synthetic data:

Smart meter measurements from GOFLEX...

... on a standard distribution grid.



https://goflex-project.eu/

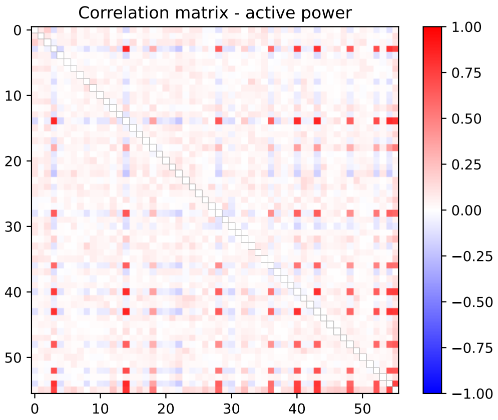
Khan and Hayes, *2022 IEEE PESGM* (2022), doi: 10.1109/PESGM48719.2022.9916806

Issue...

Park et al. (2018) require

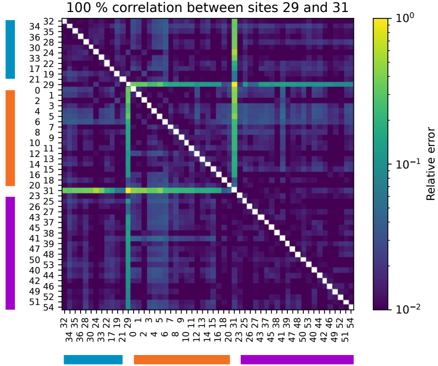
$$\mathbb{E}(P_j P_k) = 0\,,$$

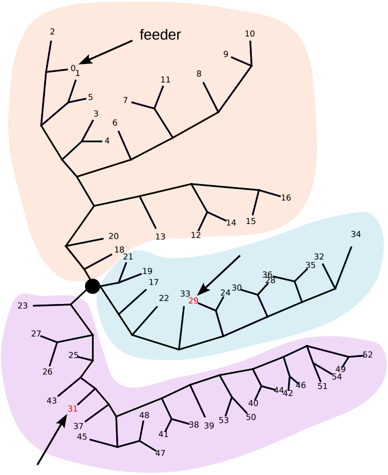
which is not always true.



Let's go back to synthetic data

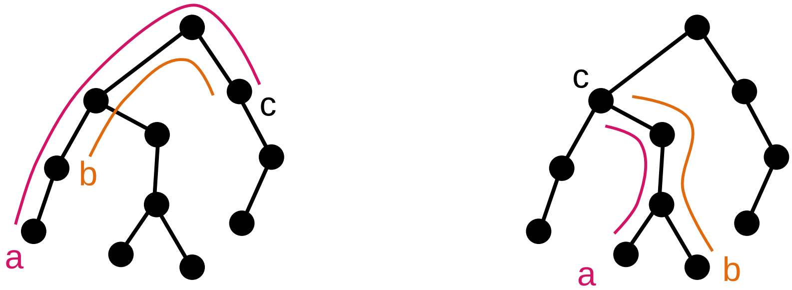
Same grid, but Gaussian time series with controlled correlations.



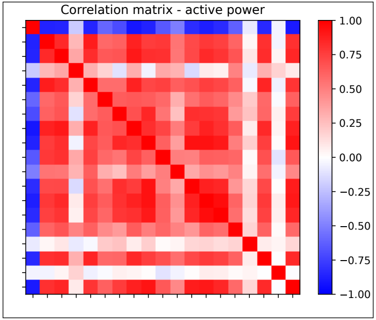


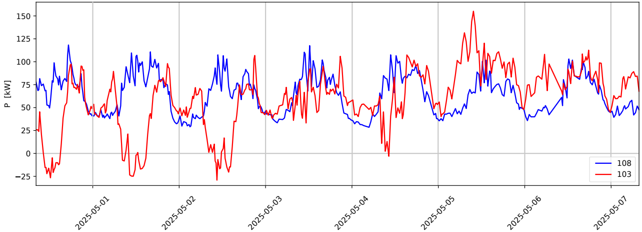
There is hope...

Recursive grouping algorithm



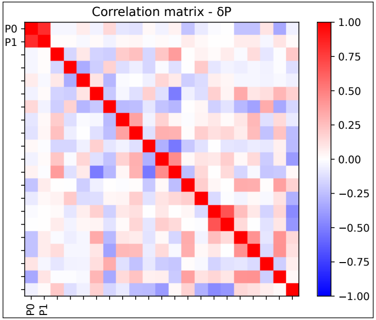
(Un)luckily...

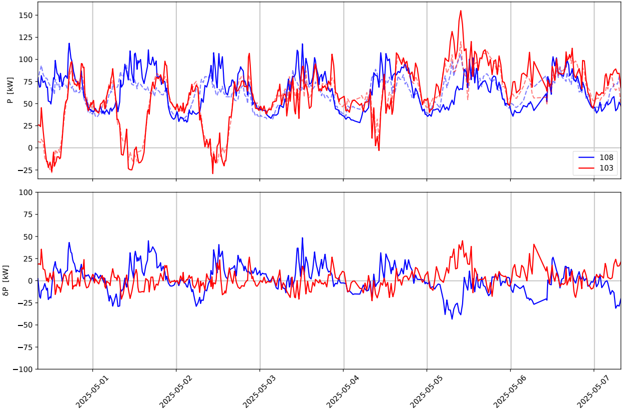




https://grid2050.ch/

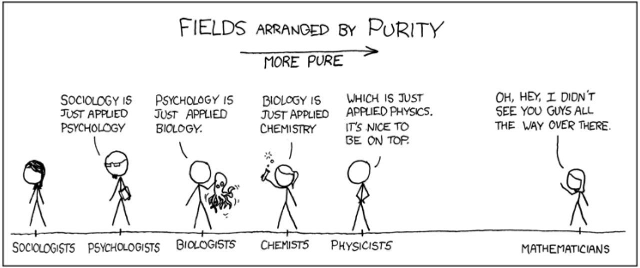
Work in progress...





https://grid2050.ch/

Conclusion



Main lesson learned:

- We are working at a sensitive spot in terms of SNR.
- Data are hard to find actually!

https://xkcd.com/435/

Correlated noise

Idea: Extract information from the noise.

Correlated noise

Idea: Extract information from the noise.

Consider a time-correlated noise $\boldsymbol{\xi}$:

$$\mathbb{E}[\xi_j(t)] = 0 \qquad \mathbb{E}[\xi_j(t)\xi_k(t')] = \delta_{jk} e^{-\tau_0|t-t'|}$$

Measure its impact at the vertices $\mathbf{x}$ (actually we look at $\dot{\mathbf{x}}$).

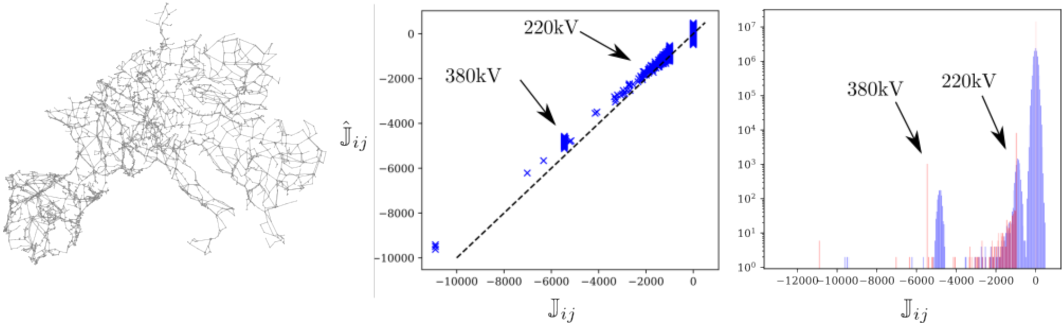
Two-point correlators

The Jacobian's ℓ-th eigenvalue and eigenvector are λ_ℓ and $\mathbf{u}^{(\ell)}$.

Then for $\lambda_\ell \tau_0 < 1$,

$$
\begin{aligned}
\mathbb{E}[\dot{x}_j \dot{x}_k] &= \delta_{jk} - \sum_{\ell=2}^{n} \mathbf{u}_j^{(\ell)} \mathbf{u}_k^{(\ell)} \frac{\lambda_\ell \tau_0}{1 + \lambda_\ell \tau_0} = \delta_{jk} - \sum_{\ell=2}^{n} \mathbf{u}_j^{(\ell)} \mathbf{u}_k^{(\ell)} \lambda_\ell \tau_0 \sum_{m=0}^{\infty} (-\lambda_\ell \tau_0)^m \\
&= \delta_{jk} + \sum_{m=1}^{\infty} (-\tau_0)^m \sum_{\ell=2}^{n} \mathbf{u}_j^{(\ell)} \lambda_\ell^m \mathbf{u}_k^{(\ell)} \\
&= \delta_{jk} + \sum_{m=1}^{\infty} (-\tau_0)^m \left(\mathcal{J}^m\right)_{jk}
\end{aligned}
$$

Results



In practice, we can only determine existence of edges, not their weight.