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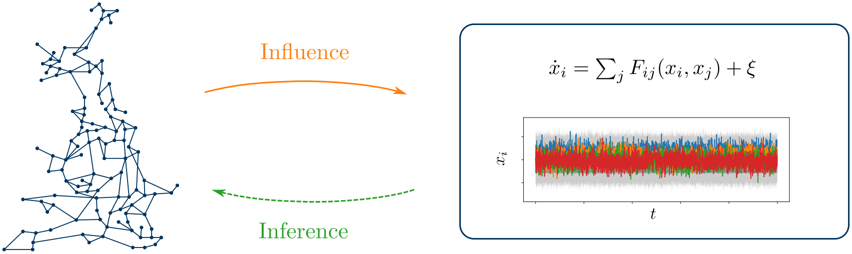
Data-driven inference of power grids: challenges and opportunities

People

The graph impacts dynamics... and vice versa



W.-X. Wang, Y.-C. Lai, and C. Grebogi, *Phys. Rep.* **644** (2016).

I. Brugere, B. Gallagher, and T. Y. Berger-Wolf, *ACM Comput. Surv.* **51** (2018).

Various approaches

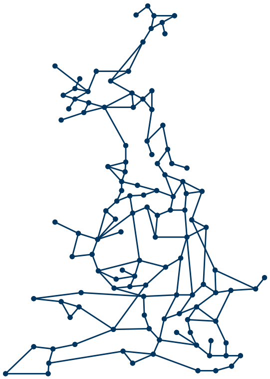
Probing: [Yu et al., *Phys. Rev. Lett.* **97** (2006)], [Timme, *Phys. Rev. Lett.* **98** (2007)], [Dong et al., *PLoS ONE* **8** (2013)], [Basiri et al., *Phys. Rev. E* **98** (2018)], [Tyloo and D., *J. Phys. Complexity* **2** (2021)], ...

Maximum likelihood/cost minimization: [Hoang et al., *Phys. Rev. E* **99** (2019)], [Makarov et al., *J. Neurosci. Methods* **144**(2005)], [Shandilya and Timme, *New J. Phys.* **13** (2011)], [Panaggio et al., *Chaos* **29** (2019)], ...

Statistical properties of trajectories: [Dahlhaus et al., *J. Neurosci. Methods* **77** (1997)], [Sameshima and Baccalá, *J. Neurosci. Methods* **94** (1999)], [Ren et al., *Phys. Rev. Lett.* **104** (2010)], [Newman, *Nature Physics* **14** (2018)], [Peixoto, *Phys. Rev. Lett.* **123** (2019)], ...

Notations

A graph	$\mathcal{G} = (V, E)$
The adjacency matrix	$A \in \mathbb{R}^{n \times n}$
The degree matrix	$D \in \mathbb{R}^{n \times n}$
The Laplacian matrix	$L = D - A$



Power grid analysis

In AC power grids, all quantities become complex:

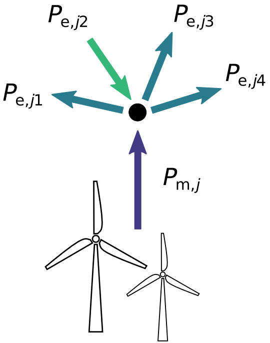
- Voltage: $\underline{V}_j = V_j e^{i\omega_j}$
- Current: $\underline{I}_j = I_j e^{i\phi_j}$
- Power: $S_j = P_j + iQ_j$
- Impedance: $Z_{jk} = R_{jk} + iX_{jk}$
- Admittance: $Y_{jk} = 1/Z_{jk} = G_{jk} + iB_{jk}$

Power flow equations

Power flow equations:

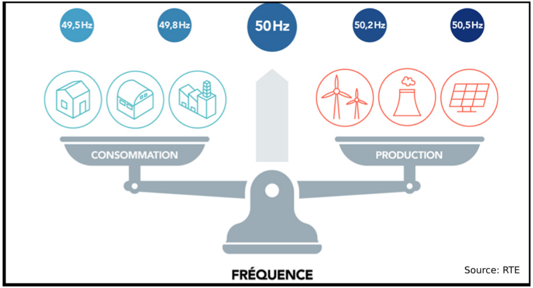
$$P_j = \sum_k \underbrace{V_j V_k \left[B_{jk} \sin(\theta_j - \theta_k) + G_{jk} \cos(\theta_j - \theta_k)\right]}_{P_{\mathrm{e},jk}}$$

$$Q_j = \sum_k V_j V_k \left[B_{jk} \cos(\theta_j - \theta_k) - G_{jk} \sin(\theta_j - \theta_k)\right]$$



J. Machowski, J. W. Bialek, and J. R. Bumby, *Power System Dynamics*, 2nd ed. (Wiley, Chichester, U.K, 2008).

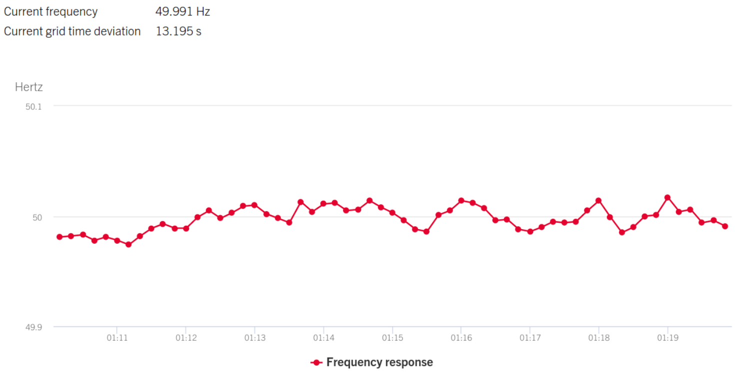
The swing equations



$$\dot{\theta}_j = P_j - \sum_k P_{\mathrm{e},jk}$$

J. Machowski, J. W. Bialek, and J. R. Bumby, *Power System Dynamics*, 2nd ed. (Wiley, Chichester, U.K, 2008).

Grid frequency



www.swissgrid.ch

Transmission grid inference

Melvyn Tyloo
Uni Exeter

Philippe Jacquod
HES-SO Sion

doi.org/10.1063/5.0058739

Simplified problem

$$P_j = \sum_k V_j V_k \left[B_{jk} \sin(\theta_j - \theta_k) + G_{jk} \cos(\theta_j - \theta_k)\right]$$

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$$P_j = \sum_k V_j V_k \left[B_{jk} \sin(\theta_j - \theta_k) + G_{jk} \cos(\theta_j - \theta_k)\right]$$

In transmission grids (high voltage):

$$\begin{aligned} V_j &\approx \text{constant} \\ G_{jk} &\approx 0 \\ P_{\mathrm{m},j} &= \sum_k B_{jk} \sin(\theta_j - \theta_k) \end{aligned}$$

Simplifed problem (bis)

The swing equations look like the Kuramoto model:

$$\dot{\theta}_j = P_j - \sum_k B_{jk} \sin(\theta_j - \theta_k)$$

S. H. Strogatz, Physica D **143** (2000)

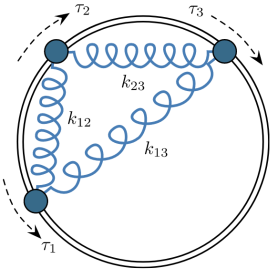
F. Dörfler and F. Bullo, Automatica **50** (2014)

Simplifed problem (bis)

The swing equations look like the Kuramoto model:

$$\dot{\theta}_j = P_j - \sum_k B_{jk} \sin(\theta_j - \theta_k)$$

Most likely converges to a steady state $\boldsymbol{\theta}^*$.



S. H. Strogatz, Physica D **143** (2000)

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Network inference from ambient noise

Considering deviations from the fixed point

$$\mathbf{x}(t) = \boldsymbol{\theta}(t) - \boldsymbol{\theta}^*$$

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Considering deviations from the fixed point

$$\mathbf{x}(t) = \boldsymbol{\theta}(t) - \boldsymbol{\theta}^*$$

The dynamics is approximated as

$$\dot{\mathbf{x}} \approx \mathcal{J}\mathbf{x} + \boldsymbol{\xi}$$

Remark: The Jacobian $\mathcal{J}$ is a (weighted) Laplacian matrix, in particular symmetric.

Correlated noise

Idea: Extract information from the noise.

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Consider a time-correlated noise $\boldsymbol{\xi}$:

$$\mathbb{E}[\xi_j(t)] = 0$$
$$\mathbb{E}[\xi_j(t)\xi_k(t')] = \delta_{jk} e^{-\tau_0 |t-t'|}$$

Measure its impact at the vertices $\mathbf{x}$ (actually we look at $\dot{\mathbf{x}}$).

Two-point correlators

The Jacobian's ℓ-th eigenvalue and eigenvector are λ_ℓ and $\mathbf{u}^{(\ell)}$.

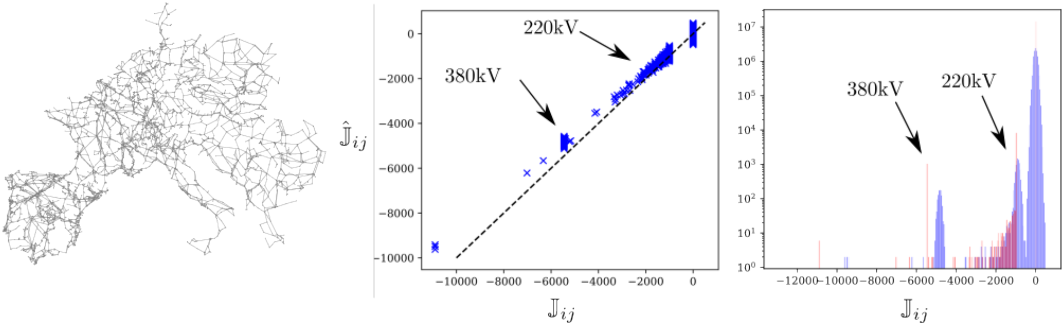
Then for $\lambda_\ell \tau_0 < 1$,

$$\mathbb{E}[\dot{x}_j \dot{x}_k] = \delta_{jk} - \sum_{\ell=2}^{n} \mathbf{u}_j^{(\ell)} \mathbf{u}_k^{(\ell)} \frac{\lambda_\ell \tau_0}{1 + \lambda_\ell \tau_0} = \delta_{jk} - \sum_{\ell=2}^{n} \mathbf{u}_j^{(\ell)} \mathbf{u}_k^{(\ell)} \lambda_\ell \tau_0 \sum_{m=0}^{\infty} (-\lambda_\ell \tau_0)^m$$

$$= \delta_{jk} + \sum_{m=1}^{\infty} (-\tau_0)^m \sum_{\ell=2}^{n} \mathbf{u}_j^{(\ell)} \lambda_\ell^m \mathbf{u}_k^{(\ell)}$$

$$= \delta_{jk} + \sum_{m=1}^{\infty} (-\tau_0)^m \left(\mathcal{J}^m\right)_{jk}$$

Results



In practice, we can only determine existence of edges, not their weight.

Distribution grid inference

Marc Gillioz
HES-SO Sion

Distribution grid vs. transmission grid

Relevance:

- Line parameters are often not known.
- Sometimes even the structure is not clear.
- Grid operators are interested in accurate models.
- By 2027, 80% of deployment.

Pros:

- Structure is usually simple (tree).

Cons:

- Actual data availability.
- Data quality.

One among many algorithms

Under **"reasonable"** assumptions, Park, Deka, and Chertkov (2018) can estimate the inverse of the resistance and reactance Laplacian matrices: $L_r^\dagger$ and $L_x^\dagger$.

$$\mathbb{E}(V_j P_k) = (L_r^\dagger)_{jk}\mathbb{E}(P_k^2) + (L_x^\dagger)_{jk}\mathbb{E}(P_k Q_k)$$
$$\mathbb{E}(V_j Q_k) = (L_r^\dagger)_{jk}\mathbb{E}(P_k Q_k) + (L_x^\dagger)_{jk}\mathbb{E}(Q_k^2)$$

S. Park, D. Deka, and M. Chertkov, Proceeding of PSCC (2018)

Effective resistance

Hence, we get the **effective resistance**:

$$d_r(i,j) = (L^\dagger)_{ii} + (L^\dagger)_{jj} - 2(L^\dagger)_{ij}$$

which is a distance.
As the graph is a tree,

$$\Omega_{ij} = \sum_{e \in P_{ij}} R_e$$

Recursive grouping algorithm

Given pairwise distances $d(a, b)$ and assuming **the graph is a tree**. Compute

$$\Phi_{abc} = d(a, c) - d(b, c)$$

Then

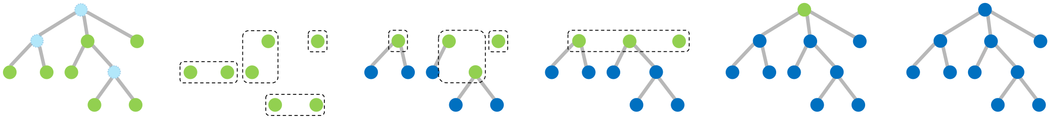
1. If $d(a, b) = \Phi_{abc}$ for all c ,then $b \to a$.
2. If $\Phi_{abc} = \Phi_{abc'}$ for all c, c', then $a \leftrightarrow b$.

In case 2., a new parent h is added. Then

$$d(a, h) = \frac{1}{2}(d(a, b) + \Phi_{abc}) \qquad d(c, h) = d(a, c) - d(a, h)$$

D. J. Klein and M. Randić, J. Math. Chem. **12** (1993)

Recursive grouping algorithm



S. Park, D. Deka, and M. Chertkov, Proceeding of PSCC (2018)

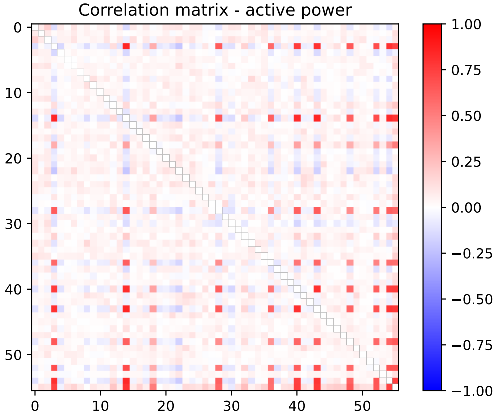
Issue...

Park et al. (2018) require

$$\mathbb{E}(P_j P_k) = 0\,,$$

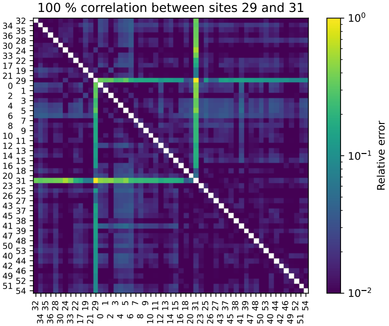
which is not always true.

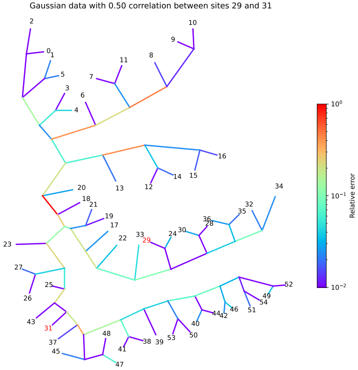
Idea: Sub-sampling



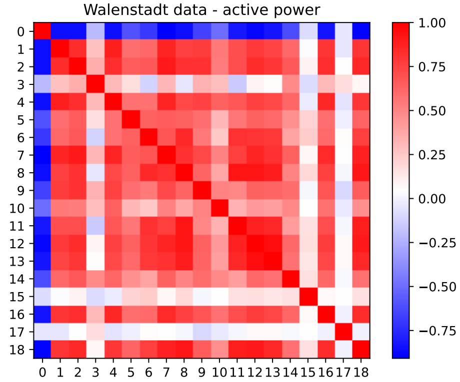
Our hope

Correlation does not impedes all resistances.





Unfortunately...



Conclusion

Correlations carry information about the underlying network...

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but too much correlation impedes inference.

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Thank you!

GeoCoW 2026

Geometry of Complex Webs

November 1 - 6, 2026

Les Diablerets

https://swissmaprs.ch/events/geocow-geometry-of-complex-webs/